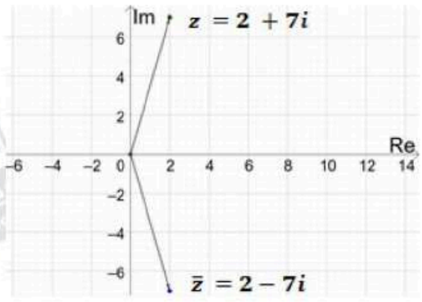
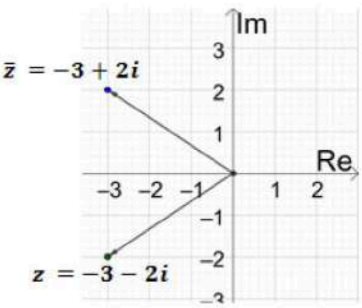
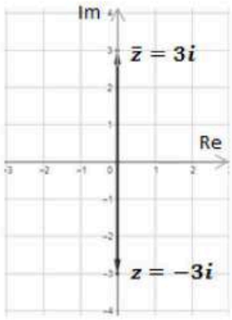
إجابات كتاب الطالب– مادة الرياضيات– الصف الثاني الثانوي العلمي ف1

الوحدة الثالثة: الأعداد المركبة

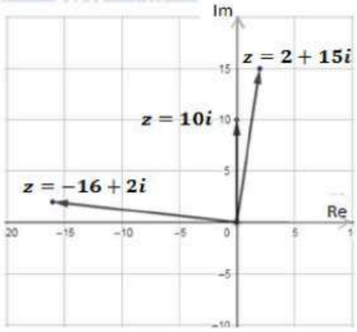
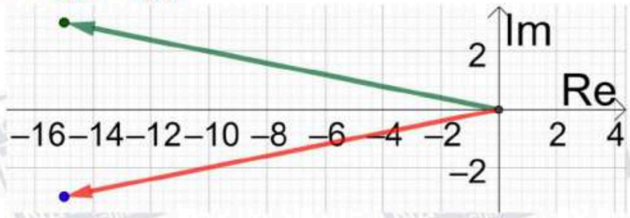
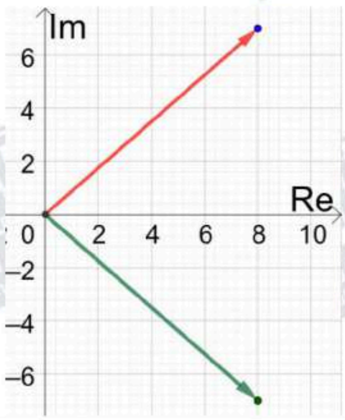
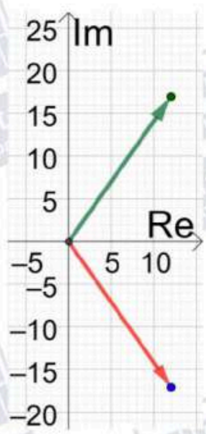
الدرس الأول: الأعداد المركبة

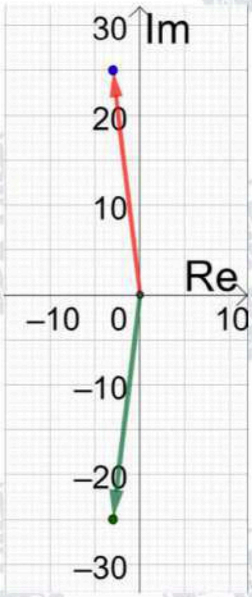
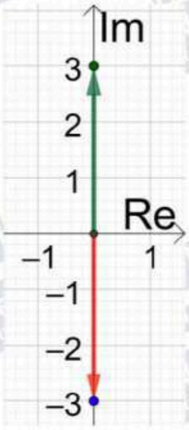
	مسألة اليوم صفحة 140
	إذا تصورنا وجود جذر تربيعي للعدد -1 في مجموعة من مجموعات الأعداد، فإن: $\left(\sqrt{-1}\right)^2 + 1 = -1 + 1 = 0$ وبالتالي يكون $\sqrt{-1}$ حلًّا للمعادلة $x^2 + 1 = 0$
	أتحقق من فهمي صفحة 141
a	$\sqrt{-75} = \sqrt{-1 \times 25 \times 3} = \sqrt{-1} \times \sqrt{25} \times \sqrt{3} = 5i\sqrt{3}$
b	$\sqrt{-49} = \sqrt{-1 \times 49} = \sqrt{-1} \times \sqrt{49} = 7i$
	أتحقق من فهمي صفحة 142
a	$\sqrt{-27} \times \sqrt{-48} = \sqrt{-1 \times 27} \times \sqrt{-1 \times 48}$ $= i\sqrt{9 \times 3} \times i\sqrt{16 \times 3}$ $= i^2\sqrt{9 \times 3 \times 16 \times 3}$ $= 36i^2 = -36$
b	$\sqrt{-50} \times -4i = \sqrt{-1 \times 50} \times (-4i)$ $= 5i\sqrt{2} \times (-4i) = -20\sqrt{2}i^2 = 20\sqrt{2}$
c	$i^{2021} = (i^2)^{1010} \times i = (-1)^{1010} \times i = i$
	أتحقق من فهمي صفحة 144
	$x + 5 + (4y - 9)i = 12 - 5i \rightarrow x + 5 = 12$ و $4y - 9 = -5$ $\rightarrow x = 7 , y = 1$

	أتحقق من فهمي صفحة 145
a	$z = 2 + 7i$, $\bar{z} = 2 - 7i$ 
b	$z = -3 - 2i$, $\bar{z} = -3 + 2i$ 
c	$z = -3i$, $\bar{z} = 3i$ 
	أتحقق من فهمي صفحة 146
a	$z = -3 - 6i\sqrt{2} \rightarrow \|z\| = \sqrt{(-3)^2 + \left(-6\sqrt{2}\right)^2} = \sqrt{81} = 9$
b	$z = -2i \rightarrow \|z\| = \sqrt{(0)^2 + (-2)^2} = \sqrt{4} = 2$
c	$z = 4 + \sqrt{-20} = 4 + \sqrt{-1} \times \sqrt{20} = 4 + i\sqrt{20}$ $\rightarrow \|z\| = \sqrt{(4)^2 + \left(\sqrt{20}\right)^2} = \sqrt{36} = 6$

	أتحقق من فهمي صفحة 150
a	$z = 8 + 2i$ $Arg(z) = \tan^{-1}\left(\frac{2}{8}\right) \approx 0.24$
b	$z = -5 + 12i$ $Arg(z) = \pi - \tan^{-1}\left(\frac{12}{5}\right) \approx 1.97$
c	$z = -2 - 3i$ $Arg(z) = -\left(\pi - \tan^{-1}\left(\frac{3}{2}\right)\right) \approx -2.16$
d	$z = 8 - 8i\sqrt{3}$ $Arg(z) = -\tan^{-1}\left(\frac{8\sqrt{3}}{8}\right) \approx -\frac{\pi}{3}$
	أتحقق من فهمي صفحة 152
a	$\|z\| = 4\sqrt{2}, Arg(z) = -\frac{3\pi}{4}$ $z = r(\cos\theta + i\sin\theta) = 4\sqrt{2}\left(\cos\left(-\frac{3\pi}{4}\right) + i\sin\left(-\frac{3\pi}{4}\right)\right)$
b	$z = -4 - 4i$ $\rightarrow r = \|z\| = \sqrt{(-4)^2 + (-4)^2} = 4\sqrt{2}$ $Arg(z) = -\left(\pi - \tan^{-1}\left(\frac{4}{4}\right)\right) \approx -\frac{3\pi}{4}$ $z = r(\cos\theta + i\sin\theta) = 4\sqrt{2}\left(\cos\left(-\frac{3\pi}{4}\right) + i\sin\left(-\frac{3\pi}{4}\right)\right)$
c	$z = 2i$ $\rightarrow r = \|z\| = \sqrt{(0)^2 + (2)^2} = 2$ $Arg(z) = \frac{\pi}{2}$ $z = r(\cos\theta + i\sin\theta) = 2\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right)$
	أتدرب وأحل المسائل صفحة 152
1	$\sqrt{-19} = \sqrt{-1 \times 19} = \sqrt{-1} \times \sqrt{19} = i\sqrt{19}$

2	$\sqrt{-\frac{12}{25}} = \sqrt{-1 \times \frac{12}{25}} = \sqrt{-1} \times \sqrt{\frac{12}{25}} = \frac{2\sqrt{3}}{5} i$
3	$\sqrt{-\frac{9}{32}} = \sqrt{-1 \times \frac{9}{32}} = \sqrt{-1} \times \sqrt{\frac{9}{32}} = \frac{3}{4\sqrt{2}} i$
4	$\sqrt{-53} = \sqrt{-1 \times 53} = \sqrt{-1} \times \sqrt{53} = i\sqrt{53}$
5	$i^{26} = (i^2)^{13} = -1$
6	$i^{39} = (i^2)^{19} \times i = (-1)^{19} \times i = -i$
7	$(i)(2i)(-7i) = (2i^2)(-7i) = (-2)(-7i) = 14i$
8	$\sqrt{-6} \times \sqrt{-6} = \sqrt{-1 \times 6} \times \sqrt{-1 \times 6}$ $= i\sqrt{6} \times i\sqrt{6}$ $= 6i^2 = -6$
9	$\sqrt{-4} \times \sqrt{-8} = \sqrt{-1 \times 4} \times \sqrt{-1 \times 8}$ $= 2i \times 2\sqrt{2}i$ $= 4\sqrt{2}i^2 = -4\sqrt{2}$
10	$2i \times \sqrt{-9} = 2i \times \sqrt{-1 \times 9}$ $= 2i \times 3i$ $= 6i^2 = -6$
11	$\frac{2 + \sqrt{-4}}{2} = \frac{2 + 2i}{2} = 1 + i$
12	$\frac{8 + \sqrt{-16}}{2} = \frac{8 + 4i}{2} = 4 + 2i$
13	$\frac{10 - \sqrt{-50}}{5} = \frac{10 - 5i\sqrt{2}}{5} = 2 - i\sqrt{2}$

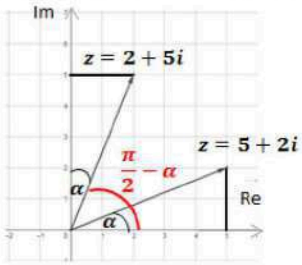
14	$z = 2 + 15i$ $\rightarrow Re(z) = 2, Im(z) = 15$	
15	$z = 10i$ $\rightarrow Re(z) = 0, Im(z) = 10$	
16	$z = -16 - 2i$ $\rightarrow Re(z) = -16, Im(z) = -2$	
17	$z = -15 + 3i$, $\bar{z} = -15 - 3i$ 	
18	$z = 8 - 7i$, $\bar{z} = 8 + 7i$ 	
19	$z = 12 + 17i$, $\bar{z} = 12 - 17i$ 	

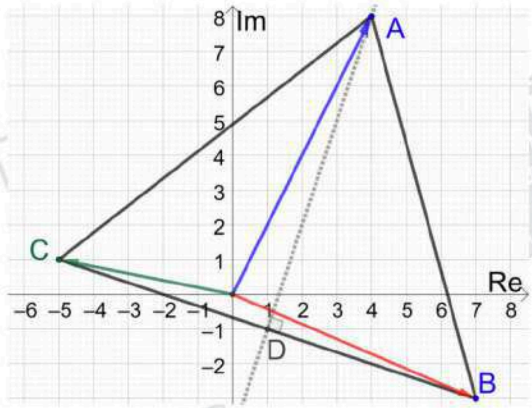
20	$z = -3 - 25i$, $\bar{z} = -3 + 25i$ 
21	$z = 3i$, $\bar{z} = -3i$ 
22	$z = 15$, $\bar{z} = 15$ 
23	$z = -5 + 5i$ $\bar{z} = -5 - 5i$ $\lvert z \rvert = \sqrt{(-5)^2 + (5)^2} = 5\sqrt{2}$

24	$z = 3 + 3\sqrt{3}i$ $\bar{z} = 3 - 3\sqrt{3}i$ $\lvert z \rvert = \sqrt{(3)^2 + (3\sqrt{3})^2} = \sqrt{9+27} = 6$
25	$z = 6 - 8i$ $\bar{z} = 6 + 8i$ $\lvert z \rvert = \sqrt{(6)^2 + (-8)^2} = \sqrt{36+64} = 10$
26	$x^2 - 1 + i(2y-5) = 8 + 9i \quad \rightarrow x^2 - 1 = 8$ و $2y - 5 = 9$ $\rightarrow x = \pm 3$ و $y = 7$
27	$2x + 3y + i(x - 2y) = 8 - 3i \quad \rightarrow 2x + 3y = 8$ و $x - 2y = -3$ $\rightarrow x = 1$ و $y = 2$
28	$y - 3 + i(3x + 2) = 9 + i\,(y - 4) \quad \rightarrow y - 3 = 9$ و $3x + 2 = y - 4$ $\rightarrow y = 12$ و $x = 2$
29	$i(2x - 5y) + 3x + 5y = 7 + 3i \quad \rightarrow 2x - 5y = 3$ و $3x + 5y = 7$ $\rightarrow x = 2$ و $y = \frac{1}{5}$
30	$z = 1$ $Arg(z) = \tan^{-1}\left(\frac{0}{1}\right) = 0$
31	$z = 3i$ $Arg(z) = \frac{\pi}{2}$
32	$z = -5 - 5i$ $Arg(z) = -\left(\pi - \tan^{-1}\left(\frac{5}{5}\right)\right) = -\frac{3\pi}{4}$
33	$z = 1 - i\sqrt{3}$ $Arg(z) = -\tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = -\frac{\pi}{3}$

34	$z = 6\sqrt{3} + 6i$ $Arg(z) = \tan^{-1}\left(\frac{6}{6\sqrt{3}}\right) = \frac{\pi}{6}$
35	$z = 3 - 4i$ $Arg(z) = -\tan^{-1}\left(\frac{4}{3}\right) \approx -0.93$
36	$z = -12 + 5i$ $Arg(z) = \pi - \tan^{-1}\left(\frac{5}{12}\right) \approx 2.75$
37	$z = -58 - 93i$ $Arg(z) = -\left(\pi - \tan^{-1}\left(\frac{93}{58}\right)\right) \approx -2.13$
38	$z = -4 + 2i$ $Arg(z) = \pi - \tan^{-1}\left(\frac{2}{4}\right) \approx 2.68$
39	$r = \|z\| = 2$ $Arg(z) = \frac{\pi}{2}$ $z = r(\cos\theta + i\sin\theta) = 2\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right)$
40	$r = \|z\| = 3, \quad Arg(z) = \frac{\pi}{3}$ $z = r(\cos\theta + i\sin\theta) = 3\left(\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right)$
41	$r = \|z\| = 7, Arg(z) = \frac{5\pi}{6}$ $z = r(\cos\theta + i\sin\theta) = 7\left(\cos\left(\frac{5\pi}{6}\right) + i\sin\left(\frac{5\pi}{6}\right)\right)$
42	$r = \|z\| = 1, Arg(z) = \frac{\pi}{4}$ $z = r(\cos\theta + i\sin\theta) = 1\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right) = \cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)$

43	$z = 6$ $\rightarrow r = \lvert z\rvert = \sqrt{(6)^2 + (0)^2} = 6$ $Arg(z) = 0$ $z = r(\cos\theta + i\sin\theta) = 6(\cos(0) + i\sin(0))$
44	$z = 1 + i$ $\rightarrow r = \lvert z\rvert = \sqrt{(1)^2 + (1)^2} = \sqrt{2}$ $Arg(z) = \tan^{-1}\left(\frac{1}{1}\right) = \frac{\pi}{4}$ $z = r(\cos\theta + i\sin\theta) = \sqrt{2}\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right)$
45	$z_1 = 4\sqrt{3} - 4i \quad \rightarrow \overline{z_1} = 4\sqrt{3} + 4i$ $Arg(z_2) = Arg(\overline{z_1}) = \tan^{-1}\left(\frac{4}{4\sqrt{3}}\right) = \frac{\pi}{6}$ $z_2 = r(\cos\theta + i\sin\theta) = 40\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$ $= 40\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = 20\sqrt{3} + 20i$ إذن، $z_2 = 20\sqrt{3} + 20i$
46	$z = r(\cos\theta + i\sin\theta) = 10\sqrt{2}\left(\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right)$ $= 10\sqrt{2}\left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right) = -10 + 10i$ إذن، $z = -10 + 10i$
47	بما أن z في الربع الثاني إذن $\bar{z}$ في الربع الثالث  فتكون الزاوية بينهما هي $\frac{\pi}{2}$

48	$z=-8+8i$ $\lvert z\rvert=\sqrt{(-8)^2+(8)^2}=8\sqrt{2}$
49	$Arg(z)=\pi-\tan^{-1}\left(\frac{8}{8}\right)=\frac{3\pi}{4}$
50	$\lvert\bar{z}\rvert=\lvert z\rvert=8\sqrt{2}$
51	$\bar{z}=-8-8i \quad \rightarrow Arg(\bar{z})=-\left(\pi-\tan^{-1}\left(\frac{8}{8}\right)\right)=-\frac{3\pi}{4}$ أو نكتب مباشرة: $Arg(\bar{z})=-Arg(z)=-\frac{3\pi}{4}$
52	$Arg(5+2i)=\alpha=\tan^{-1}\left(\frac{2}{5}\right)$ $Arg(-5-2i)=-\left(\pi-\tan^{-1}\left(\frac{2}{5}\right)\right)=-(\pi-\alpha)=-\pi+\alpha$
53	$Arg(5-2i)=-\tan^{-1}\left(\frac{2}{5}\right)=-\alpha$
54	$Arg(-5+2i)=\pi-\tan^{-1}\left(\frac{2}{5}\right)=\pi-\alpha$
55	 يوضح الرسم المجاور العلاقة بين سعة كل من العددين $z=5+2i$ ، و $z=2+2i$ $Arg(2+5i)=\tan^{-1}\left(\frac{5}{2}\right)=\frac{\pi}{2}-\tan^{-1}\left(\frac{2}{5}\right)=\frac{\pi}{2}-\alpha$
56	$Arg(-2+5i)=\pi-\tan^{-1}\left(\frac{5}{2}\right)=\pi-\left(\frac{\pi}{2}-\alpha\right)=\frac{\pi}{2}+\alpha$
57	$z=5+im \quad , \qquad \lvert z\rvert=6, \qquad 0<Arg(z)<\frac{\pi}{2}$ $\lvert z\rvert=\sqrt{(5)^2+(m)^2}=\sqrt{25+m^2}=6 \quad \rightarrow 25+m^2=36 \quad \rightarrow m=\pm\sqrt{11}$ لكن $0<Arg(z)<\frac{\pi}{2}$ وهذا يعني أن z في الربع الأول، ومنه $m=\sqrt{11}$

58	$z = 5 + 3ik \quad , \quad \lvert z\rvert = 13$ $\lvert z\rvert = \sqrt{(5)^2 + (3k)^2} = \sqrt{25 + 9k^2} = 13 \quad \rightarrow \quad 25 + 9k^2 = 169 \quad \rightarrow k = \pm 4$
59	$\lvert z_1\rvert = r = 4\sqrt{5}, \qquad Arg(z_1) = \tan^{-1}(2) = \theta$ (نستنتج هنا أن z_1 يقع في الربع الأول، ففي الأرباع الأخرى تكون السعة بإشارة سالبة أو تحتوي π) $\tan\theta = 2 \quad \rightarrow \sin\theta = \frac{2}{\sqrt{5}} \ , \cos\theta = \frac{1}{\sqrt{5}}$ $z_1 = r(\cos\theta + i\sin\theta) = 4\sqrt{5}(\cos\theta + i\sin\theta) = 4\sqrt{5}\left(\frac{1}{\sqrt{5}} + i\frac{2}{\sqrt{5}}\right) = 4 + 8i$
60	$z_1 = 4 + 8i\ , z_2 = 7 - 3i\ , z_3 = -5 + i$  $AC = \sqrt{(4-(-5))^2 + (8-1)^2}$ $= \sqrt{130}$ $AB = \sqrt{(4-7)^2 + (8-(-3))^2}$ $= \sqrt{130}$ $BC = \sqrt{(7-(-5))^2 + (-3-1)^2}$ $= \sqrt{160}$ ومنه فإن المثلث ABC متطابق الضلعين، نتّخذ BC قاعدةً له ونجد إحداثيي النقطة D نقطة منتصف القاعدة BC: $D\left(\frac{7-5}{2}, \frac{-3+1}{2}\right) \rightarrow D(1,-1)$ ارتفاع هذا المثلث هو القطعة المستقيمة الواصلة بين الرأس ومنتصف القاعدة وهو $\overline{AD}$ $AD = \sqrt{(4-1)^2 + (8--1)^2} = \sqrt{90}$ لتكن مساحة المثلث ABCهي Aفإن: $A = \frac{1}{2} \times \sqrt{160} \times \sqrt{90} = 60$ إذن، مساحة المثلث ABC تساوي 60 وحدة مربعة.