



إجابات كتاب الطالب - مادة الرياضيات - الصف الثاني الثانوي الأدبي ف1

الوحدة الثانية: التفاضل

الدرس الأول: قاعدة السلسلة

مسألة اليوم صفحة 54

$$N(t) = 20 - \frac{30}{\sqrt{9-t^2}}$$
$$N'(t) = \frac{30 \left(\frac{-2t}{2\sqrt{9-t^2}} \right)}{9-t^2} = \frac{-30t}{(9-t^2)\sqrt{9-t^2}}$$

أتحقق من فهمي صفحة 56

$$y = (x^2 - 2)^4$$
$$u = x^2 - 2$$
$$y = u^4$$
$$\frac{du}{dx} = 2x$$
$$\frac{dy}{du} = 4u^3$$
$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$
$$= 4u^3 \times 2x$$
$$= 8xu^3$$
$$= 8x(x^2 - 2)^3$$



b	$y = \sqrt{x^3 + 4x} = (x^3 + 4x)^{\frac{1}{2}}$ $u = x^3 + 4x$ $y = u^{\frac{1}{2}}$ $\frac{du}{dx} = 3x^2 + 4$ $\frac{dy}{du} = \frac{1}{2} u^{-\frac{1}{2}}$ $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ $= \frac{1}{2} u^{-\frac{1}{2}} \times (3x^2 + 4)$ $= \frac{3x^2 + 4}{2\sqrt{x^3 + 4x}}$
	أتحقق من فهمي صفحة 58
a	$f'(x) = 5(x^4 + 1)^4(4x^3)$ $= 20x^3(x^4 + 1)^4$ $f'(1) = 20(1)^3((1)^4 + 1)^4 = 20 \times 16 = 320$
b	$f(x) = \sqrt{x^2 + 3x + 2} = (x^2 + 3x + 2)^{\frac{1}{2}}$ $f'(x) = \frac{1}{2}(x^2 + 3x + 2)^{-\frac{1}{2}}(2x + 3)$ $= \frac{1}{2}(2x + 3)(x^2 + 3x + 2)^{-\frac{1}{2}}$ $= \frac{2x + 3}{2\sqrt{x^2 + 3x + 2}}$ $f'(2) = \frac{2(2) + 3}{2\sqrt{2^2 + 3 \times 2 + 2}} = \frac{7}{2\sqrt{12}}$
c	$f(x) = \sqrt[4]{(2x^2 - 7)^5} = (2x^2 - 7)^{\frac{5}{4}}$ $f'(x) = \frac{5}{4}(2x^2 - 7)^{\frac{1}{4}}(4x)$ $= \frac{5}{4}(4x)(2x^2 - 7)^{\frac{1}{4}}$ $= 5x \times \sqrt[4]{2x^2 - 7}$ $f'(4) = 5 \times 4 \times \sqrt[4]{2(4)^2 - 7} = 20\sqrt[4]{25}$



أتحقق من فهمي صفحة 59

a

$$\begin{aligned}f'(x) &= 4(1 + x^3)^3(3x^2) + 8x^7 \\ &= 12x^2(1 + x^3)^3 + 8x^7\end{aligned}$$

b

$$\begin{aligned}f(x) &= (2x - 1)^{\frac{1}{3}} - (x - 3)^3 \\ f'(x) &= \frac{1}{3}(2x - 1)^{-\frac{2}{3}}(2) - 3(x - 3)^2(1) \\ &= \frac{2}{3\sqrt[3]{(2x - 1)^2}} - 3(x - 3)^2\end{aligned}$$

أتحقق من فهمي صفحة 61

a

$$P'(t) = \frac{20t + 1}{2\sqrt{10t^2 + t + 229}}$$

b

$$t = 2020 - 2015 = 5$$

$$P'(5) = \frac{101}{2\sqrt{250 + 5 + 229}} = \frac{101}{2\sqrt{484}} = \frac{101}{2 \times 22} = \frac{101}{44} \approx 2.3$$

إذن، في سنة 2020 يزداد إجمالي الأرباح بمعدل 2300 دينار لكل سنة.



أتحقق من فهمي صفحة 62

$$\begin{aligned}\frac{dy}{du} &= 5u^4 + 3u^2 \\ \frac{du}{dx} &= -4 \\ \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ &= (5u^4 + 3u^2) \times -4 \\ &= -4(5(3-4x)^4 + 3(3-4x)^2) \\ &= -20(3-4x)^4 - 12(3-4x)^2 \\ \left. \frac{dy}{dx} \right|_{x=2} &= -20(625) - 12(25) = -12800\end{aligned}$$

أدرب وأحل المسائل صفحة 62

1

$$\begin{aligned}f'(x) &= 4(1+2x)^3(2) \\ &= 8(1+2x)^3\end{aligned}$$

2

$$\begin{aligned}f'(x) &= -5(3-2x^2)^{-6}(-4x) \\ &= 20x(3-2x^2)^{-6} \\ &= \frac{20x}{(3-2x^2)^6}\end{aligned}$$

3

$$\begin{aligned}f'(x) &= \frac{3}{2}(x^2-7x+1)^{\frac{1}{2}}(2x-7) \\ &= \frac{3}{2}(2x-7)\sqrt{x^2-7x+1}\end{aligned}$$

4

$$f'(x) = \frac{-1}{2\sqrt{7-x}}$$

5

$$\begin{aligned}f'(x) &= 16(2+8x)^3(8) \\ &= 128(2+8x)^3\end{aligned}$$

6

$$\begin{aligned}f(x) &= (4x-8)^{-\frac{1}{3}} \\ f'(x) &= -\frac{1}{3}(4x-8)^{-\frac{4}{3}}(4) \\ &= -\frac{4}{3}(4x-8)^{-\frac{4}{3}} \\ &= \frac{-4}{3\sqrt[3]{(4x-8)^4}}\end{aligned}$$



7	$f'(x) = \frac{9x^2}{2\sqrt{5+3x^3}}$
8	$f'(x) = \frac{1}{2\sqrt{x}} + 2(x-3)$
9	$f(x) = (2x - x^5)^{\frac{1}{3}} + (4-x)^2$ $f'(x) = \frac{1}{3}(2x - x^5)^{-\frac{2}{3}}(2 - 5x^4) + 2(4-x)(-1)$ $= \frac{2 - 5x^4}{3\sqrt[3]{(2x - x^5)^2}} - 8 + 2x$
10	$f'(x) = 4(\sqrt{x} + 5)^3 \times \frac{1}{2\sqrt{x}}$ $= \frac{2(\sqrt{x} + 5)^3}{\sqrt{x}}$
11	$f'(x) = \frac{3(2x-5)^2(2)}{2\sqrt{(2x-5)^3}}$ $= \frac{3(2x-5)^2}{\sqrt{(2x-5)^3}} = 3\sqrt{2x-5}$
12	$f'(x) = 5(2x^3 - 3x^2 + 4x + 1)^4(6x^2 - 6x + 4)$
13	$f(x) = (4x+1)^{-2}$ $f'(x) = -2(4x+1)^{-3}(4)$ $= -\frac{8}{(4x+1)^3}$ $f'\left(\frac{1}{4}\right) = -\frac{8}{\left(4 \times \frac{1}{4} + 1\right)^3} = -1$
14	$f'(x) = \frac{-x}{\sqrt{25-x^2}}$ $f'(3) = \frac{-3}{\sqrt{25-(3)^2}} = -\frac{3}{4}$



15 $\frac{dy}{du} = 10u + 3$

$$\frac{du}{dx} = 3x^2$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= (10u + 3) \times 3x^2$$

$$= (10(x^3 + 1) + 3) \times 3x^2$$

$$= (10x^3 + 13) \times 3x^2$$

$$= 30x^5 + 39x^2$$

16 $y = (2u + 5)^{\frac{1}{3}}$

$$\frac{dy}{du} = \frac{1}{3} (2u + 5)^{-\frac{2}{3}} (2) = \frac{2}{3} (2u + 5)^{-\frac{2}{3}}$$

$$\frac{du}{dx} = 2x - 1$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= \frac{2}{3} (2u + 5)^{-\frac{2}{3}} \times (2x - 1)$$

$$= \frac{2}{3} (2(x^2 - x) + 5)^{-\frac{2}{3}} \times (2x - 1)$$

$$= \frac{4x - 2}{3\sqrt[3]{(2x^2 - 2x + 5)^2}}$$



17	$\frac{dy}{du} = 6u - 5$ $\frac{du}{dx} = 2x$ $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ $= (6u - 5) \times (2x)$ $= (6(x^2 - 1) - 5) \times (2x)$ $\left. \frac{dy}{dx} \right _{x=2} = (6(4 - 1) - 5) \times (4) = 52$
18	$\frac{dy}{du} = 3(1 + u^2)^2(2u) = 6u(1 + u^2)^2$ $\frac{du}{dx} = 2$ $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ $= 6u(1 + u^2)^2 \times (2)$ $= 12(2x - 1)(1 + (2x - 1)^2)^2$ $\left. \frac{dy}{dx} \right _{x=1} = 12(2 - 1)(1 + (2 - 1)^2)^2 = 48$
19	$C'(x) = \frac{1000(2x - 0.1)}{2\sqrt{x^2 - 0.1x}} = \frac{1000x - 50}{\sqrt{x^2 - 0.1x}}$
20	$C'(20) = \frac{1000(20) - 50}{\sqrt{(20)^2 - 0.1(20)}} = \frac{19950}{\sqrt{398}} \approx 1000$
21	$N(t) = 400(1 - 3(t^2 + 2)^{-2})$ $N'(t) = 400 \left(6(t^2 + 2)^{-3}(2t) \right) = \frac{4800t}{(t^2 + 2)^3}$ $N'(1) = \frac{4800}{(1 + 2)^3} \approx 178$



22	$N'(4) = \frac{4800(4)}{(16+2)^3} \approx 3$
23	$f'(x) = g'(h(x)) \times h'(x)$ $f'(3) = g'(h(3)) \times h'(3)$ $= g'(2) \times -2$ $= 6 \times -2 = -12$
24	$f'(x) = 3(h(x))^2 \times h'(x)$ $f'(3) = 3(h(3))^2 \times h'(3)$ $= 3(2)^2 \times -2 = -24$
25	$h'(x) = f'(g(x)) \times g'(x)$ $h'(2) = f'(g(2)) \times g'(2)$ $= f'(3) \times -1$ نجد مشتقة f ونحسب $f'(3)$ $f(u) = u^2 - 1 \rightarrow f'(u) = 2u \rightarrow f'(3) = 2 \times 3 = 6$ إذن، $h'(2) = f'(3) \times -1$ $= 6 \times -1 = -6$
26	$y = (x^2 - 4)^5$ $0 = (x^2 - 4)^5 \rightarrow x^2 - 4 = 0 \rightarrow (x - 2)(x + 2) = 0$ $\rightarrow x = 2 \text{ or } x = -2$ $\frac{dy}{dx} = 5(x^2 - 4)^4(2x) = 10x(x^2 - 4)^4$ $\left. \frac{dy}{dx} \right _{x=2} = 10(2)(2^2 - 4)^4 = 0$ $\left. \frac{dy}{dx} \right _{x=-2} = 10(-2)((-2)^2 - 4)^4 = 0$
27	$p(x)$ هو الاقتران الوحيد الذي يمكن اشتقاقه بدون تطبيق قاعدة السلسلة



$$28 \quad f(x) = (2x + (x^2 + x)^4)^{\frac{1}{3}}$$

$$f'(x) = \frac{1}{3} (2x + (x^2 + x)^4)^{-\frac{2}{3}} (2 + 4(x^2 + x)^3(2x + 1))$$

$$= \frac{2 + 4(x^2 + x)^3(2x + 1)}{3\sqrt[3]{(2x + (x^2 + x)^4)^2}}$$

الدرس الثاني: مشتقا ضرب والقسمه

مسألة اليوم صفحة 64

$$\frac{dh}{dt} = \frac{(8 + t^3)(3t^2) - (t^3)(3t^2)}{(8 + t^3)^2}$$

$$= \frac{24t^2 + 3t^5 - 3t^5}{(8 + t^3)^2}$$

$$= \frac{24t^2}{(8 + t^3)^2}$$

أتحقق من فهمي صفحة 65

$$a \quad f'(x) = (x^3 + 4)(14x - 4) + (7x^2 - 4x)(3x^2)$$

$$= 14x^4 - 4x^3 + 56x - 16 + 21x^4 - 12x^3$$

$$= 35x^4 - 16x^3 + 56x - 16$$

$$b \quad f'(x) = (\sqrt{x} + 1)(3) + (3x - 2)\left(\frac{1}{2\sqrt{x}}\right)$$

$$= 3\sqrt{x} + 3 + \frac{3x}{2\sqrt{x}} - \frac{1}{\sqrt{x}}$$

$$= 3\sqrt{x} + 3 + \frac{3}{2}\sqrt{x} - \frac{1}{\sqrt{x}}$$

$$= \frac{9}{2}\sqrt{x} + 3 - \frac{1}{\sqrt{x}}$$



أتحقق من فهمي صفحة 67

a

$$\begin{aligned}f'(x) &= \frac{(x-2)(3) - (3x+1)(1)}{(x-2)^2} \\&= \frac{3x-6-3x-1}{(x-2)^2} \\&= \frac{-7}{(x-2)^2}\end{aligned}$$

b

$$\begin{aligned}f'(x) &= \frac{(x^2+1)(-3x^{-4}) - (x^{-3})(2x)}{(x^2+1)^2} \\&= \frac{-3x^{-2} - 3x^{-4} - 2x^{-2}}{(x^2+1)^2} \\&= \frac{-5x^{-2} - 3x^{-4}}{(x^2+1)^2}\end{aligned}$$

أتحقق من فهمي صفحة 68

a

$$\begin{aligned}P'(t) &= \frac{(2t^2+9)(0) - (5)(4t)}{(2t^2+9)^2} \\&= \frac{-20t}{(2t^2+9)^2}\end{aligned}$$

b

$$P'(2) = \frac{-40}{(8+9)^2} = \frac{-40}{289} \approx -0.14$$

يتناقص عدد السكان بمعدل 140 نسمة لكل سنة بعد سنتين من الآن

أتحقق من فهمي صفحة 70

a

$$\begin{aligned}f'(x) &= \frac{-(1)(-3x^2)}{(1-x^3)^2} \\&= \frac{3x^2}{(1-x^3)^2}\end{aligned}$$



b	$f'(x) = \frac{-(3)(2)}{(2x+1)^2}$ $= \frac{-6}{(2x+1)^2}$
أتحقق من فهمي صفحة 71	
a	$f'(x) = (20x) \times 6(4x^3 - 1)^5(12x^2) + (4x^3 - 1)^6(20)$ $= (4x^3 - 1)^5(1520x^3 - 20)$
b	$f'(x) = \frac{(x+2)^4(2x) - (x^2-1) \times 4(x+2)^3 \times 1}{(x+2)^8}$ $= \frac{2x(x+2)^4 - 4(x^2-1)(x+2)^3}{(x+2)^8}$ $= \frac{(x+2)^3(2x(x+2) - 4(x^2-1))}{(x+2)^8}$ $= \frac{-2x^2 + 4x + 4}{(x+2)^5}$
أدرب وأحل المسائل صفحة 71	
1	$f'(x) = x \times 5(1+3x)^4(3) + (1+3x)^5(1)$ $= (1+3x)^4(18x+1)$
2	$f'(x) = \frac{(x+1)(1) - (x+3)(1)}{(x+1)^2}$ $= \frac{-2}{(x+1)^2}$
3	$f'(x) = (2x+1)^5 \times 4(3x+2)^3(3) + (3x+2)^4 \times 5(2x+1)^4 \times 2$ $= 2(2x+1)^4(3x+2)^3(27x+16)$



4	$f'(x) = \frac{(2x-1)^2(6x) - (3x^2) \times 2(2x-1)(2)}{(2x-1)^4}$ $= \frac{6(2x-1)(2x^2 - x - 2x^2)}{(2x-1)^4}$ $= \frac{-6x}{(2x-1)^3}$
5	$f'(x) = \frac{(\sqrt{5x+3})(6) - (6x)\left(\frac{5}{2\sqrt{5x+3}}\right)}{5x+3} = \frac{30x+18-15x}{(5x+3)\sqrt{5x+3}}$ $= \frac{15x+18}{(5x+3)\sqrt{5x+3}}$
6	$f'(x) = (4x-1)(2x) + (x^2-5)(4)$ $= 8x^2 - 2x + 4x^2 - 20$ $= 12x^2 - 2x - 20$
7	$f'(x) = \frac{(2x-7)(2x) - (x^2+6)(2)}{(2x-7)^2}$ $= \frac{4x^2 - 14x - 2x^2 - 12}{(2x-7)^2}$ $= \frac{2x^2 - 14x - 12}{(2x-7)^2}$
8	$f'(x) = \frac{(1+\sqrt{x})(1) - (x)\left(\frac{1}{2\sqrt{x}}\right)}{(1+\sqrt{x})^2}$ $= \frac{1+\sqrt{x} - \frac{1}{2}\sqrt{x}}{(1+\sqrt{x})^2}$ $= \frac{1+\frac{1}{2}\sqrt{x}}{(1+\sqrt{x})^2}$



9	$f'(x) = (x+1) \times \frac{1}{2\sqrt{x-1}} + (\sqrt{x-1})(1)$ $= \frac{x+1}{2\sqrt{x-1}} + \sqrt{x-1} = \frac{x+1+2x-2}{2\sqrt{x-1}} = \frac{3x-1}{2\sqrt{x-1}}$
10	$f'(x) = \frac{(1)(5+2x) - (x)(2)}{(5+2x)^2} - 8x^3$ $= \frac{5}{(5+2x)^2} - 8x^3$
11	$f'(x) = \frac{(-5)(2)(x+2)(1)}{(x+2)^4}$ $= \frac{-10}{(x+2)^3}$
12	$f'(x) = \left(x + \frac{2}{x}\right)(2x) + (x^2 - 3)\left(1 - \frac{2}{x^2}\right)$ $= 2x^2 + 4 + x^2 - 3 - 2 + \frac{6}{x^2}$ $= 3x^2 - 1 + \frac{6}{x^2}$
13	$f'(x) = (8x + \sqrt{x})(10x) + (5x^2 + 3)\left(8 + \frac{1}{2\sqrt{x}}\right)$ $= 80x^2 + 10x^{\frac{3}{2}} + 40x^2 + \frac{5}{2}x^{\frac{3}{2}} + 24 + \frac{3}{2\sqrt{x}}$ $= 120x^2 + \frac{25}{2}x^{\frac{3}{2}} + 24 + \frac{3}{2\sqrt{x}}$
14	$f(x) = 5x - 25 + 50x^{-2} - 10x^{-3}$ $f'(x) = 5 - 100x^{-3} + 30x^{-4}$
15	$f'(x) = (x^2) \times 3(3x-1)^2 \times 3 + (3x-1)^3(2x)$ $f'(1) = (1)3(3-1)^2 \times 3 + (3(1)-1)^3(2(1)) = 36 + 16 = 52$



16	$f'(x) = (3x) \left(\frac{-1}{2\sqrt{5-x}} \right) + (\sqrt{5-x})(3)$ $f'(4) = \frac{(3 \times 4)(-1)}{2\sqrt{5-4}} + (\sqrt{5-4})(3)$ $= \frac{-12}{2 \times 1} + 1 \times 3 = -6 + 3 = -3$
17	$f'(x) = \frac{(2x+1)(1) - (x-1)(2)}{(2x+1)^2} = \frac{3}{(2x+1)^2}$ $f'(2) = \frac{3}{(4+1)^2} = \frac{3}{25}$
18	$f'(x) = (2x+3) \times 2(x-2)(1) + (x-2)^2(2)$ $f'(0) = 3 \times 2(-2) + 2(-2^2) = -12 + 8 = -4$
19	$S'(t) = \frac{(4+0.3t)(2000) - 2000t(0.3)}{(4+0.3t)^2}$ $= \frac{8000}{(4+0.3t)^2}$
20	$t = 2030 - 2020 = 10$ $S'(10) = \frac{8000}{(4+3)^2} = \frac{8000}{49} \approx 163$ يتزايد إجمالي المبيعات بمقدار 163 ألف دينار لكل سنة في عام 2030م.
21	<u>ملاحظة:</u> نرجو حذف كلمة (بالآلاف) من مقدمة السؤال لتدل P على عدد السكان بوحدة الفرد الواحد (شخص أو نسمة). $P'(t) = 12(2t^2 + 100)(1) + (t+20) \times 12(4t) = 12(6t^2 + 80t + 100)$
22	$P'(6) = 12(216 + 480 + 100) = 12(796) = 9552$ يتزايد عدد السكان بمعدل 9552 نسمة كل سنة بعد 6 سنوات من الآن.



23

$$M'(t) = \frac{(t + 1.9)(5.8) - (5.8t)(1)}{(t + 1.9)^2}$$

$$= \frac{11.02}{(t + 1.9)^2}$$

$$M'(5) = \frac{11.02}{(5 + 1.9)^2} \approx 0.23$$

$$\frac{dy}{du} = u \times 3(u^2 + 3)^2(2u) + (u^2 + 3)^3(1) = (u^2 + 3)^2(7u^2 + 3)$$

$$\frac{du}{dx} = 2(x + 3)(1) = 2x + 6$$

عندما $x = -2$ ، فإن $u = (-2 + 3)^2 = 1$

24

$$\left. \frac{dy}{dx} \right|_{x=-2} = \left. \frac{dy}{du} \right|_{u=1} \times \left. \frac{du}{dx} \right|_{x=-2}$$

$$\left. \frac{dy}{du} \right|_{u=1} = (1^2 + 3)^2(7(1^2) + 3) = 16(10) = 160$$

$$\left. \frac{du}{dx} \right|_{x=-2} = 2(-2) + 6 = 2$$

$$\left. \frac{dy}{dx} \right|_{x=-2} = 160 \times 2 = 320$$

$$\frac{dy}{du} = \frac{(u + 1) \times 3u^2 - u^3(1)}{(u + 1)^2} = \frac{2u^3 + 3u^2}{(u + 1)^2}$$

$$\frac{du}{dx} = 3(x^2 + 1)^2(2x) = 6x(x^2 + 1)^2$$

عندما $x = 1$ ، فإن $u = (1^2 + 1)^3 = 8$

25

$$\left. \frac{dy}{dx} \right|_{x=1} = \left. \frac{dy}{du} \right|_{u=8} \times \left. \frac{du}{dx} \right|_{x=1}$$

$$\left. \frac{dy}{du} \right|_{u=8} = \frac{2(8^3) + 3(8^2)}{(8 + 1)^2} = \frac{1216}{81}$$

$$\left. \frac{du}{dx} \right|_{x=1} = 6(1)(1^2 + 1)^2 = 24$$

$$\left. \frac{dy}{dx} \right|_{x=1} = \frac{1216}{81} \times 24 = \frac{9728}{27}$$



26	$(fg)'(x) = (f \times g)'(x)$ $= f(x) \times g'(x) + g(x) \times f'(x)$ $(fg)'(2) = f(2) \times g'(2) + g(2) \times f'(2)$ $= 4 \times 2 + 3 \times -1 = 5$
27	$\left(\frac{f}{g}\right)'(x) = \frac{g(x) \times f'(x) - f(x) \times g'(x)}{(g(x))^2}$ $\left(\frac{f}{g}\right)'(2) = \frac{g(2) \times f'(2) - f(2) \times g'(2)}{(g(2))^2} = \frac{3 \times -1 - 4 \times 2}{(3)^2} = -\frac{11}{9}$
28	$(3f + fg)'(x) = 3f'(x) + f(x) \times g'(x) + g(x) \times f'(x)$ $(3f + fg)'(2) = 3f'(2) + f(2) \times g'(2) + g(2) \times f'(2)$ $= 3 \times -1 + 4 \times 2 + 3 \times -1 = 2$
29	$f'(x) = (x(4x-3)^6) \times 9(1-4x)^8(-4) + (1-4x)^9 \times (x \times 6(4x-3)^5(4) + (4x-3)^6 \times (1))$ $f'(x) = -36x(4x-3)^6(1-4x)^8 + (1-4x)^9(24x(4x-3)^5 + (4x-3)^6)$ $= (4x-3)^5(1-4x)^8(-36x(4x-3) + (1-4x)(24x+4x-3))$ $= (4x-3)^5(1-4x)^8(-256x^2 + 148x - 3)$
30	$f(x) = \frac{2x}{x+5} + \frac{6x}{x^2+7x+10}$ $= \frac{2x}{x+5} + \frac{6x}{(x+5)(x+2)}$ $= \frac{2x(x+2)}{(x+5)(x+2)} + \frac{6x}{(x+5)(x+2)}$ $= \frac{2x^2+10x}{(x+5)(x+2)}$ $= \frac{2x(x+5)}{(x+5)(x+2)}$ $= \frac{2x}{x+2}$
31	$f'(x) = \frac{(x+2)(2) - (2x)(1)}{(x+2)^2} = \frac{4}{(x+2)^2}$ $f'(3) = \frac{4}{(3+2)^2} = \frac{4}{25}$



$$f'(x) = \frac{(\sqrt{x})(2) - (2x + 8)\left(\frac{1}{2\sqrt{x}}\right)}{x}$$

$$0 = \frac{(\sqrt{x})(2) - (2x + 8)\left(\frac{1}{2\sqrt{x}}\right)}{x}$$

$$32 \quad (\sqrt{x})(2) - (2x + 8)\left(\frac{1}{2\sqrt{x}}\right) = 0$$

$$2\sqrt{x} - \sqrt{x} - \frac{4}{\sqrt{x}} = 0$$

$$\sqrt{x} - \frac{4}{\sqrt{x}} = 0$$

$$\sqrt{x} = \frac{4}{\sqrt{x}}$$

$$x = 4$$



الدرس الثالث: مشتقتا الاقتران الأسّي الطبيعي والاقتران اللوغاريتمي الطبيعي

مسألة اليوم صفحة 73

$$N = 10000(1 - e^{-0.15d})$$

$$N'(d) = 10000(0.15e^{-0.15d})$$

$$= 1500e^{-0.15d}$$

أتحقق من فهمي صفحة 74

a $f'(x) = 2e^x$

b $f'(x) = \frac{1}{3}x^{-\frac{2}{3}} + e^x = \frac{1}{3\sqrt[3]{x^2}} + e^x$

c $\frac{dy}{dx} = xe^x + e^x = e^x(x + 1)$

أتحقق من فهمي صفحة 75

a $f'(x) = 7e^{7x+1}$

b $f'(x) = 3x^2e^{x^3}$

c $f'(x) = \frac{5}{2\sqrt{x}}e^{\sqrt{x}}$

أتحقق من فهمي صفحة 76

a $P'(t) = 50(-0.004)e^{-0.004t} = -0.2e^{-0.004t}$

$$P'(500) = -0.2e^{-0.004(500)} = -0.2e^{-2} \approx -0.03$$

تتناقص الطاقة المتبقية بمعدل 0.03 واط لكل يوم بعد 500 يوم

أتحقق من فهمي صفحة 78

a $f'(x) = \frac{4}{x}$

b $f'(x) = \frac{1}{2\sqrt{x}} + \frac{1}{x}$

c $f'(x) = \frac{(x)\left(\frac{1}{x}\right) - (\ln x)(1)}{x^2} = \frac{1 - \ln x}{x^2}$



أتحقق من فهمي صفحة 80

a $f'(x) = \frac{8}{8x} = \frac{1}{x}$

b $f'(x) = 2 \times \frac{7x^6}{x^7} = \frac{14}{x}$

c $f'(x) = \frac{9}{9x+2}$

أتدرب وأحل المسائل صفحة 80

1 $f'(x) = 2e^x$

2 $f'(x) = 3e^{3x+9}$

3 $f'(x) = (x^2 + 3x - 9)(e^x) + (e^x)(2x + 3) = e^x(x^2 + 5x - 6)$

4 $f'(x) = \frac{x^4 e^x - e^x(4x^3)}{x^8} = \frac{x e^x - 4e^x}{x^5}$

5 $f'(x) = 6 \times \frac{1}{2\sqrt{x}} e^{\sqrt{x}} = \frac{3}{\sqrt{x}} e^{\sqrt{x}}$

6 $f'(x) = \frac{(1+e^x)(e^x) - e^x(e^x)}{(1+e^x)^2} = \frac{e^x}{(1+e^x)^2}$

7 $f'(x) = (e^x + 2)(e^x) + (e^x - 1)(e^x) = 2e^{2x} + e^x$

8 $f'(x) = (e^{-2x}) \times 5(2x-1)^4 \times 2 + (2x-1)^5(-2e^{-2x})$
 $= 2e^{-2x}(2x-1)^4(6-2x)$

9 $f'(x) = 3x^2 - 5 \times 2e^{2x} = 3x^2 - 10e^{2x}$

10 $f'(x) = \frac{3}{x}$

11 $f'(x) = (x^3) \left(\frac{1}{x} \right) + (\ln x)(3x^2) = x^2 + 3x^2 \ln x$

12 $f'(x) = \frac{x^2 \left(\frac{1}{x} \right) - (\ln x)(2x)}{x^4} = \frac{x - 2x \ln x}{x^4} = \frac{1 - 2 \ln x}{x^3}$



13	$f'(x) = (x^2) \left(\frac{4}{4x} \right) + (\ln(4x))(2x) = x + 2x \ln(4x)$
14	$f'(x) = \frac{\frac{(x)(1) - (x+1)(1)}{x^2}}{\frac{x+1}{x}} = \frac{\frac{-1}{x^2}}{\frac{x+1}{x}} = \frac{-1}{x^2} \times \frac{x}{x+1} = \frac{-1}{x(x+1)}$
15	$f'(x) = \frac{2x}{2\sqrt{x^2-1}} = \frac{2x}{2\sqrt{x^2-1}} \times \frac{1}{\sqrt{x^2-1}} = \frac{x}{x^2-1}$
16	$f'(x) = 4(\ln x)^3 \times \frac{1}{x} = \frac{4(\ln x)^3}{x}$
17	$f'(x) = \frac{2x}{x^2-5}$
18	$f'(x) = (x^4) \left(\frac{1}{x} \right) + (\ln x)(4x^3) - \frac{1}{2}e^x = x^3 + 4x^3 \ln x - \frac{1}{2}e^x$
19	$f'(x) = (e^{2x}) \left(\frac{1}{x} \right) + (\ln x)(2e^{2x}) = \frac{e^{2x}(1+x \ln x)}{x}$
20	$f'(x) = (\ln 3x) \left(\frac{7}{7x} \right) + (\ln 7x) \left(\frac{3}{3x} \right) = \frac{\ln 3x + \ln 7x}{x}$
21	$f'(x) = \frac{e^x}{e^x-2}$
22	$f'(x) = (e^{2x-1}) \left(\frac{2}{2x-1} \right) + (\ln(2x-1))(2e^{2x-1})$ $f'(1) = (e^{2-1}) \left(\frac{2}{2-1} \right) + (\ln(2-1))(2e^{2-1}) = 2e + 0 = 2e$
23	$f'(x) = \frac{x \left(\frac{2x}{x^2} \right) - (\ln x^2)(1)}{x^2} = \frac{2 - \ln x^2}{x^2}$ $f'(4) = \frac{2 - \ln 16}{16}$



24	$P'(t) = \frac{-100 \times -e^{3-t}}{(1 + e^{3-t})^2} = \frac{100e^{3-t}}{(1 + e^{3-t})^2}$ $P'(3) = \frac{100e^{3-3}}{(1 + e^{3-3})^2} = \frac{100}{4} = 25$
25	$m'(t) = (t) \left(\frac{1}{t} \right) + (\ln t)(1) = 1 + \ln t$
26	$\frac{dy}{du} = 2e^{2u}$ $\frac{du}{dx} = 2x$ $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ $= 2e^{2u} \times 2x$ $= 4xe^{2u}$ $= 4xe^{2(x^2+1)}$
27	$\frac{dy}{du} = \frac{1}{u+1}$ $\frac{du}{dx} = e^x$ $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ $= \frac{1}{u+1} \times e^x$ $= \frac{e^x}{e^x + 1}$
28	$\frac{dy}{dx} = \frac{k}{kx} = \frac{1}{x}$



$$\frac{dy}{dx} = \frac{(e^{3x}) \times \left(7 \times \frac{1}{x} - 3x^2\right) - (7 \ln x - x^3)(3e^{3x})}{(e^{3x})^2}$$

$$\left. \frac{dy}{dx} \right|_{x=1} = \frac{(e^3) \times (7 \times 1 - 3) - (7 \ln 1 - 1)(3e^3)}{(e^3)^2}$$

$$29 \quad = \frac{4e^3 + 3e^3}{(e^3)^2}$$

$$= \frac{7e^3}{(e^3)^2}$$

$$= \frac{7}{e^3}$$



الدرس الرابع: مشتقتا اقتران الجيب واقتان جيب التمام

مسألة اليوم صفحة 82

$$P(t) = 100 + 20 \sin 2\pi t$$

$$\frac{dP}{dt} = 40\pi \cos 2\pi t$$

أتحقق من فهمي صفحة 83

a $f'(x) = \cos x$

b $f'(x) = 3 + \sin x$

c $f'(x) = 3 \cos x - 2 \sin x$

أتحقق من فهمي صفحة 84

a $f'(x) = (e^x)(-\sin x) + (\cos x)(e^x) = -e^x \sin x + e^x \cos x$

b
$$\begin{aligned} f'(x) &= \frac{(\sin x)(1 - \sin x) - (x + \cos x)(\cos x)}{\sin^2 x} \\ &= \frac{\sin x - \sin^2 x - x \cos x - \cos^2 x}{\sin^2 x} \\ &= \frac{\sin x - (\sin^2 x + \cos^2 x) - x \cos x}{\sin^2 x} \\ &= \frac{\sin x - 1 - x \cos x}{\sin^2 x} \end{aligned}$$

أتحقق من فهمي مثال 3 صفحة 86

a $f'(x) = -5 \sin 5x$

b $f'(x) = \frac{\cos x}{2\sqrt{\sin x}}$

c $f'(x) = \frac{-3 \sin 3x}{\cos 3x}$



تحقق من فهمي مثال 4 صفحة 86

$$h'(x) = 4 \times \frac{\pi}{6} \cos \frac{\pi}{6} t = \frac{2\pi}{3} \cos \frac{\pi}{6} t$$

أدرب وأحل المسائل صفحة 86

1	$f'(x) = -2 \sin x + \cos x$
2	$f'(x) = -\sin x$
3	$f'(x) = \cos x + \sin x$
4	$f'(x) = (x)(\cos x) + (\sin x)(1)$ $= x \cos x + \sin x$
5	$f'(x) = (\sin x)(-\sin x) + (\cos x)(\cos x)$ $= -\sin^2 x + \cos^2 x$
6	$f'(x) = (e^x)(\cos x) + (\sin x)(e^x)$ $= e^x \cos x + e^x \sin x$
7	$f'(x) = \frac{(\cos x)(e^x) - (e^x)(-\sin x)}{\cos^2 x} = \frac{e^x \cos x + e^x \sin x}{\cos^2 x}$
8	$f'(x) = 2x \cos(x^2 + 1)$
9	$f'(x) = \frac{\cos x}{\sin x}$
10	$f'(x) = -5 \sin(5x - 2)$
11	$f'(x) = 3 \cos 3x - 6 \sin 6x$
12	$f'(x) = -(2x - 3) \sin(x^2 - 3x - 4)$
13	$f'(x) = (e^{2x})(10 \cos 10x) + (\sin 10x)(2e^{2x})$ $= 10e^{2x} \cos 10x + 2e^{2x} \sin 10x$
14	$f'(x) = (\cos x^2) \left(\frac{1}{x} \right) + (\ln x)(-2x \sin x^2)$ $= \frac{1}{x} (\cos x^2) - 2x (\ln x) \sin x^2$



15	$f'(x) = (\sqrt{x+1}) \left(\frac{\pi}{2} \cos \frac{\pi x}{2} \right) + \left(\sin \frac{\pi x}{2} \right) \left(\frac{1}{2\sqrt{x+1}} \right)$
16	$f(x) = 4(\sin x)^2$ $f'(x) = 4 \times 2(\sin x)(\cos x) = 8\sin x \cos x$
17	$f(x) = (\cos 2x)^3(\cos x)$ $f'(x) = (\cos 2x)^3(-\sin x) + (\cos x) \times 3(\cos 2x)^2 \times -2 \sin 2x$ $= -(\cos 2x)^3(\sin x) - 6(\cos x)(\cos 2x)^2 \sin 2x$
18	$f'(x) = 5 \times \frac{1}{2\sqrt{x}} \cos \sqrt{x} = \frac{5}{2\sqrt{x}} \cos \sqrt{x}$
19	$f'(x) = 2(\cos 2x - \sin x)(-2 \sin 2x - \cos x)$
20	$f'(x) = \frac{1}{2\sqrt{x}} \cos \sqrt{x} + \frac{2 \cos 2x}{2\sqrt{\sin 2x}} = \frac{1}{2\sqrt{x}} \cos \sqrt{x} + \frac{\cos 2x}{\sqrt{\sin 2x}}$
21	$f'(x) = \frac{(\sin x) \left(2(\ln x) \times \frac{1}{x} \right) - (\ln x)^2(\cos x)}{\sin^2 x}$ $= \frac{2 \sin x \ln x - x \cos x (\ln x)^2}{x \sin^2 x}$
22	$D'(t) = 400 \times 0.4 \cos 0.4t = 160 \cos 0.4t$
23	$H'(t) = 2.4 \times \frac{2\pi}{365} \cos \left(\frac{2\pi}{365} (t - 80) \right) = \frac{4.8\pi}{365} \cos \left(\frac{2\pi}{365} (t - 80) \right)$



24	$\begin{aligned}\frac{dy}{dx} &= \frac{1}{2} \left(1 - ((\sin x)(-\sin x) + (\cos x)(\cos x)) \right) \\ &= \frac{1}{2} (1 - (-\sin^2 x + \cos^2 x)) \\ &= \frac{1}{2} (1 + \sin^2 x - \cos^2 x) \\ &= \frac{1}{2} (\sin^2 x + 1 - \cos^2 x) \\ &= \frac{1}{2} (\sin^2 x + \sin^2 x) \\ &= \frac{1}{2} (2 \sin^2 x) \\ &= \sin^2 x\end{aligned}$
25	$\begin{aligned}f(x) &= (e^x \cos x)(\sin x)^2 \\ f'(x) &= (e^x \cos x)(2(\sin x)^1 \cos x) + (\sin x)^2((e^x)(-\sin x) + (\cos x)(e^x)) \\ &= e^x \sin x (2\cos^2 x - \sin^2 x + \cos x \sin x)\end{aligned}$
26	$f'(x) = -\frac{1}{x^2} \cos\left(\frac{1}{x}\right)$